# Testing Newton's second law in an accelerating system

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Newton's second law claims that force is the product of mass and acceleration ($\sum F = ma$). It could be of great use when calculating the acceleration, and thus velocity and position, of a system based on the external forces acting on it; therefore we tested its validity. Using a cart and pulley system, we examined the relationships between force, mass, and acceleration. The observed system accelerations for different values of force and mass were consistent with Newton's second law.

## I. INTRODUCTION

Force is the product of mass and acceleration:

$$\sum F = ma, \tag{1}$$

where $\sum F$ is the sum of the external forces acting on the system in N, $m$ is the mass in kg, and $a$ is the acceleration in $\mathrm{m\,s^{-2}}$. (1) illustrates how a system's forces depend on the object's mass and acceleration.

This relationship between force, mass, and acceleration is useful in determining the acceleration that acts on an object without access to information that can be used to calculate acceleration using kinematics equations (eqs 2, 3, and 4) like initial ($v_0$) and final velocity ($v_f$) measured in $\mathrm{m\,s^{-1}}$, time ($t$) measured in s, and initial ($x_0$) and final position ($x_f$) measured in m.

$$x_f = x_0 + v_0 t + \frac{1}{2}at^2 \tag{2}$$
$$v_f = v_0 + at \tag{3}$$
$$v_f^2 = v_0^2 + 2a(x_f - x_0) \tag{4}$$

To facilitate hypotheses, we set up a system with a cart on a track attached by a string and pulley to a hanging mass (depicted in Fig. 1), allowing force and mass to be varied somewhat independently since some of the mass in the system was subject to gravitational force, while some were not.

We considered the acceleration, force, and mass, and hypothesized that there could be no acceleration in the system. Newton's first law gives us the static case where

$$H_0 : \sum F = 0. \tag{5}$$

Alternatively, we hypothesized that the net force would increase as the mass increased and that the acceleration of the system would decrease as mass increased, therefore the force would increase as the acceleration increased, while acceleration and mass have an inverse relationship (6):

$$H_1 : \sum F = ma. \tag{6}$$

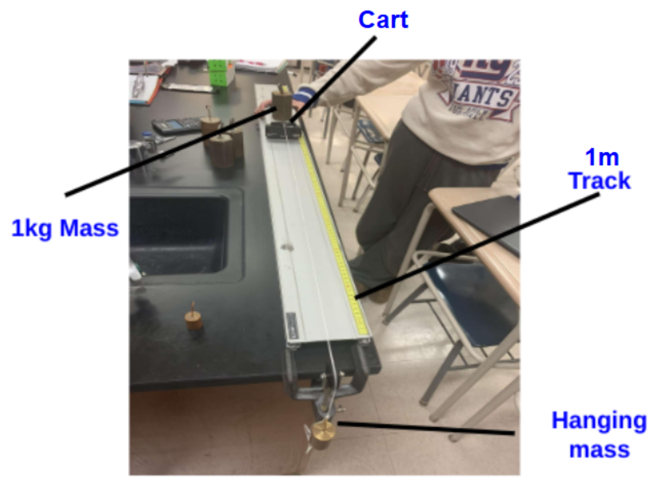


FIG. 1. Track, cart, and pulley system used for experiments. Total track length 1.0m.

Or, we hypothesized that either Newton's laws wouldn't apply, or something was erroneous with the considered forces, resulting in the force being equal to un-modeled forces acting that have a significant effect on acceleration (7) and (8).

$$H_2 : \sum F \neq 0, \tag{7}$$
$$H_3 : \sum F \neq ma. \tag{8}$$

These hypotheses were tested through a total of six trials, with a cart that had a constant weight, and different masses on the other end of the pulley. The time was measured to calculate the relationships between force, mass, and acceleration for both values of hanging mass.

## II. METHODS AND MATERIALS

### A. Tests

Tests ($n = 6$) were conducted using a track, cart, and pulley system (Fig. 1). The system included a 1.0 m

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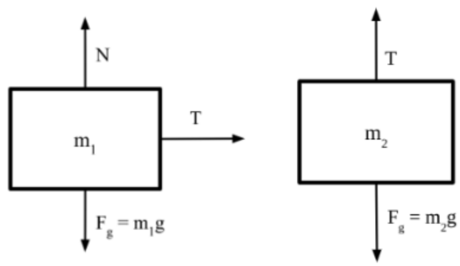


FIG. 2. Free body diagrams for $m_1$ (left) and $m_2$ (right) created in Google Drawings.

aluminum test track (Pasco Scientific; Roseville, CA) clamped to a table. The system was outfitted with a wheeled cart (Pasco Scientific; Roseville, CA) with ball bearings and knife-edge wheels. The cart's mass was 0.493 kg and it carried a 1.000 kg mass for a total mass of 1.493 kg. Attached to the cart was a string that looped over a pulley clamped to the table. On the other side of the string, for the first three trials, was a 0.050 kg mass, which was swapped out for a 0.200 kg mass for the last three trials. The hanging masses provided gravitational force to drive the system. The cart was released from rest and allowed to accelerate. We measured the time it took for the cart to move from rest 0.70 m along the track. Data were logged in a Google Document (Google; Mountain View, CA) on a school-issued Google Chromebook (Google; Mountain View, CA).

### B. Acceleration calculations

The acceleration predicted under Newton's second law (1) can be calculated using the tensions of each mass. As seen in Fig. 2, the mass on the track moving horizontally is $m_1$ and the mass falling is $m_2$. The track is frictionless, so the force acting on $m_1$ equals the tension and is calculated as

$$F_1 = m_1 a = T. \tag{9}$$

The tension is also acting upward on $m_2$, yielding

$$F_2 = -m_2 a = T - m_2 g. \tag{10}$$

Combining (9) and (10) and manipulating gives the system acceleration $a$ as a function of gravitational acceleration $g$ and the known masses $m_1$ and $m_2$:

$$a = \frac{m_2}{m_1 + m_2 + m_c} g. \tag{11}$$

$m_c = 0.493\,\mathrm{kg}$ accounts for the empty mass of the cart. Comparison of this estimate for $a$ to measured values allows us to test the validity of (1).

The measured time data were used to calculate acceleration via kinematics assuming uniform motion [1]:

$$x = x_0 + v_0 t + \frac{1}{2} a t^2. \tag{12}$$

Selecting $x_0 = 0$ and recognizing $v_0 = 0$ when starting from rest, (12) can be solved for acceleration:

$$a_{meas} = \frac{2x}{t^2}, \tag{13}$$

where $x = 0.70\,\mathrm{m}$ is the length of the track, and $t$ is measured during each trial.

## III. RESULTS

Table I shows the measured time $t$ for the system to move from rest to 0.7 m as well as the resulting acceleration calculated via (13), as hanging mass $m_2$ was varied between 0.050 kg to 0.200 kg. Total cart mass $m_1 + m_c = 1.493\,\mathrm{kg}$. All values are listed as mean ± one standard deviation with $n = 3$ replicates for each value of $m_2$.

TABLE I. Measured time $t$ for the system to move from rest to 0.7 m as well as the resulting acceleration calculated via (13), as hanging mass $m_2$ was varied between 0.050 kg to 0.200 kg. Total cart mass $m_1 + m_c = 1.493\,\mathrm{kg}$. All values are listed as mean ± one standard deviation with $n = 3$ replicates for each value of $m_2$.

| $m_2$ (kg) | $t$ (s) | $a_{meas}$ ($\mathrm{m\,s^{-2}}$) |
|---|---|---|
| 0.050 | $2.31 \pm 0.10$ | $0.26 \pm 0.02$ |
| 0.200 | $1.00 \pm 0.16$ | $1.46 \pm 0.41$ |

Fig. 3 shows the measured acceleration for each trial (13) compared to the system acceleration predicted using (11). Measured values are in good agreement with predictions based on (13) and (1).

## IV. DISCUSSION

### A. Can we confirm that $\sum F = ma$ through experimentation?

Our trials demonstrated that as the hanging mass $m_2$, or the resulting gravitational force exerted on the system, increases, the system's acceleration increases. This supported our hypothesis $\sum F = ma$ (6). The force acting on the system increased by increasing the hanging mass $m_2$. The experimental data corroborated the hypothesis because the increased pulley mass and acceleration illustrate a direct relationship between force and acceleration. Our data showed a consistent pattern where an increase in the mass of the pulley increased the net force creating

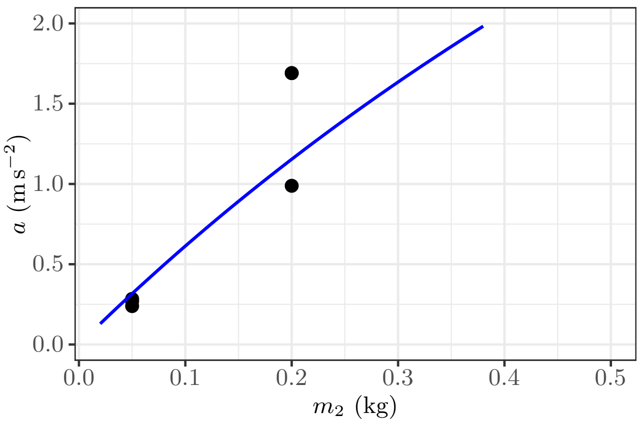


FIG. 3. Measured acceleration ($a_{meas}$) for each of the three trials, from (13) and Table I is shown by black dots for hanging mass $m_2 = 0.050$ kg and 0.200 kg. Predicted acceleration $a$ based on (11) is shown by the blue line.

an increase in acceleration. This suggests that acceleration and force are proportional, in the case that mass is held constant (6). In our experiments, total system mass was $m_1 + m_2 + m_c$, which increased slightly, resulting in the curve of Fig. 3.

### B. Sources of experimental error

A potential source of experimental error stems from the movement of the track between trials. This caused slight differences in tension, potentially affecting the calculated acceleration of the cart. Additionally, the person's timing was not consistent across all trials, nor were they randomized or rotated methodically by trial. This could result in small inconsistencies with the timing that may skew the results of calculating acceleration using a formula involving time or any other calculations involving time. In further testing, the time can be measured with sensors or videos to be more accurate. Furthermore, the expected acceleration was calculated assuming the string was massless and that friction had no effect on the cart. Since the experiment used a string with mass and since a frictionless system is impossible to achieve practically, this could result in possible differences between the experimental and expected acceleration.

## V. ACKNOWLEDGMENTS

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PR wrote the abstract, introduction, test description, part of the discussion, and recorded data during the trials. MA created graphs of the results and helped describe them and draw conclusions. CB made the free-body diagrams and momentum track diagrams, computed acceleration, and wrote aspects of the discussion and parts of other sections. KM wrote the discussion and part of the results, noted the materials, timed the trials, and recorded the results. JC wrote about the sources of the experimental error and helped collect data and the abstract. GN led the track's setup, wrote out the procedures, and conducted each trial when we collected data. Everyone contributed to editing, proofreading, and assisting others with the sections they worked on.

[1] P. A. Tipler and G. Mosca, *Physics for Scientists and Engineers*, 5th ed. (W H Freeman and Company, New York, 2004) .