Newton's second law as demonstrated in a cart-pulley-mass system

Vasudevan Govardhanen,* Gage Grant, Aidan Dumalagan, Tushaar Akula, Daivik Jajoo, Rohan Avalur, Adrit Sikdar, and Jake Cacciarelli
Science & Engineering Magnet Program, Manalapan High School, Englishtown, NJ 07726 USA
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We investigated the dynamics of a two-mass cart-pulley system to examine the relationship between mass distribution and acceleration in a nearly frictionless environment. Motion sensors tracked the cart's position along the track, allowing us to calculate velocity and acceleration over time. By systematically increasing the mass on the pulley, we observed corresponding increases in the cart's acceleration, enabling a comparison with theoretical predictions based on Newton's second law.

I. INTRODUCTION

Newton's second law of motion, asserts that the acceleration of an object is directly proportional to the net force acting upon it and inversely proportional to its mass [1]:

$$F = ma \tag{1}$$

where F is the net force applied to the object, m is its mass, and a is the resulting acceleration.

This experiment is designed to rigorously examine (1) by analyzing the dynamics of a cart-pulley-mass system. The system consists of a cart of mass m_1 connected to a pulley with a hanging mass m_2, with m_2 generating a net force $F = m_2 g$ due to gravity. On a frictionless track, the expected acceleration a of the cart can be expressed as:

$$a_{pred} = \frac{m_2 g}{m_1 + m_2} \tag{2}$$

where $g = 9.81\,\mathrm{m\,s^{-2}}$ is the acceleration due to gravity. (2) is obtained from application of Newton's second law and assumes that friction between the cart and track and within the pulley is negligible, that the pulley is massless, and that the string is massless and stiff. We systematically vary m_2 and measure the cart's displacement over time to calculate its observed acceleration, allowing for a comparison with predicted values from (2).

We hypothesize that the net applied force F is proportional to the resulting accceleration a of the cart, consistent with (1)':

$$H_1 : a \propto F. \tag{3}$$

Alternatively, we may observe that the system acceleration a is not related to the applied force, and is instead constant, in which case we would reject Newton's second law.

$$H_0 : a = a_0 \tag{4}$$

* Contact author: 426vgovardhanen@frhsd.com

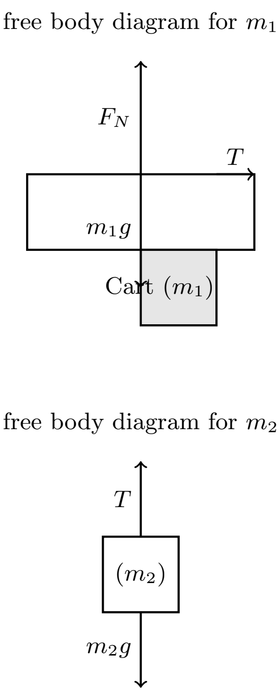

FIG. 1. Separate free body diagrams for cart m_1 and hanging mass m_2.

II. MATERIALS AND METHODS

Measurements were obtained using a small cart (PASCO Scientific; Roseville, CA) of mass $m_1 = 0.500\,\mathrm{kg}$. The cart was situated within a track (PASCO Scientific; Roseville CA) clamped to a lab bench; the cart and track were assumed to be frictionless. A string connected to the cart was hung over a pulley; the other end of the string was connected to varying masses $m_2 = 0.050\,\mathrm{kg}$, $0.070\,\mathrm{kg}$ and $0.100\,\mathrm{kg}$. m_2 was the independent variable, used to exert a varying external gravitational force $F = m_2 g$ on the system, where $g = 9.81\,\mathrm{m\,s^{-2}}$ [1]. The time t for the mass to travel distance $d = 0.50\,\mathrm{m}$ was measured by a human observer with a stopwatch with $0.01\,\mathrm{s}$ precision. One measurement was made for each value of m_2.

The measured system acceleration was then calculated

TABLE I. Measured values of t for different m_2, as well as the resulting system acceleration a, calculated using (5). $n = 1$ measurement for each value of m_2. For these measurements, $m_1 = 0.5$ kg and $d = 0.5$ m.

m_2 (kg)	t (s)	a_{meas} ($\mathrm{m\,s^{-2}}$)
0.050	2.39	0.18
0.070	2.05	0.24
0.100	1.80	0.31

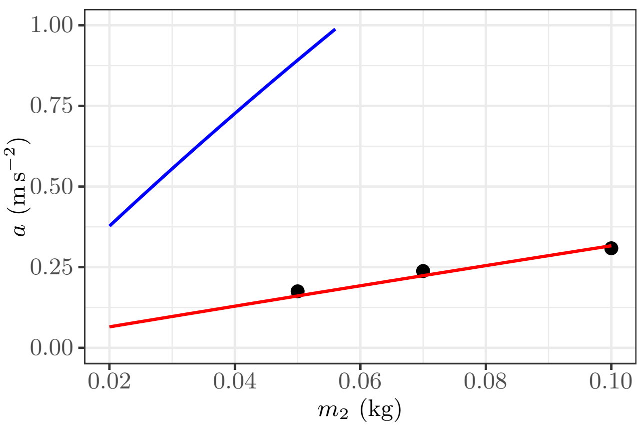

FIG. 2. Acceleration a as a function of m_2. Data from Table I. Measured values of acceleration from Table I and (5) are plotted as black dots; predictions from Newton's second law (2) shown as blue line. For these data, $m_1 = 0.500$ kg. The measured accelerations and the predictions do not agree well. The fit is much better for $m_1 = 2.5$ kg, shown as red line.

using

$$a_{meas} = \frac{2d}{t^2}. \quad (5)$$

(5) comes from simple kinematics under uniform, constant acceleration, recognizing that the system starts from rest so that $v_0 = 0$ and choosing $x_0 = 0$ [1]. The measured system acceleration from (5) can then be compared to the acceleration predicted by (2) based on Newton's second law (1).

III. RESULTS

Table I gives the measured values of t for different m_2, as well as the resulting system acceleration a. The results of Table I are also plotted in Fig. 2.

IV. DISCUSSION

The experimental results did not support the theoretical predictions of Newton's second law (see mismatch in Fig. 2). While we did observe a trend where the acceleration increased as the mass of the hanging weight increased, the predicted values of acceleration differed from those we measured by a factor of three or more. Possible explanations for this discrepancy are that we failed to correctly measure the mass of the cart or to include the dead weight of the cart or of any masses loaded into the cart; or that there is a significant amount of friction in the system. In particular, when we recalculate for $m_1 \sim 2.5$ kg we observe better agreement between the measured acceleration and those predicted by (2) (red line, Fig. 2). We also attribute our discrepancies to various real-world factors such as friction between the cart and track, which was assumed to be negligible in the idealized theoretical model, mass of the pulley and friction within it, and other experimental limitations such as timing errors or air resistance.

V. ACKNOWLEDGEMENTS

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[1] P. A. Tipler and G. Mosca, *Physics for Scientists and Engineers*, 5th ed. (W H Freeman and Company, New York, 2004) .